Essays on Social Networks and Time Series with Structural Breaks

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Ph.D. Oral Defense

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Outline

- **Chapter 1:** Estimating Peer Effects Using Partial Network
- **Chapter 2:** Count Data Models with Social Interactions under Rational Expectations
- **Chapter 3:** Selective linear segmentation for detecting relevant parameter changes

Chapter 1: Estimating Peer Effects Using Partial Network

Vincent Boucher & Elysée Aristide Houndetoungan

Peer Effects in Networked Economies

- Explain behavior by behavior of peers (Manski 1993).
  - E.g.: Taxation and smoking (direct and indirect peer effects).
- Huge literature following L.-F. Lee 2004 and Bramoullé, Djebbari, and Fortin 2009
  - Peer effects in education (Calvó-Armengol, Patacchini, and Zenou 2009);
  - Peer effects in the workplace (Cornelissen, Dustmann, and Schönberg 2017).

- Main assumption to estimate peer effects.
- Eliciting network data is expensive:
  1. Ask each subject to name their best friends. Using sample from the population leads to non-classical measurement error (see Chandrasekhar and Lewis 2011);
  2. E.g.: **More than 8,000 journal articles, presentations, manuscripts, books, book chapters and dissertations using Add Health data sets**;
  3. Risk of data error/incomplete data.

- Model,

$$y_i = \alpha \bar{y}_i + \mathbf{x}_i' \boldsymbol{\beta} + \varepsilon_i. \tag{1}$$

- We develop a method based on the distribution of the network.

$$\mathbf{A} = \begin{array}{c} \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{array} \begin{array}{c} \begin{array}{cccc} i_1 & i_2 & i_3 & i_4 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{pmatrix} \end{array}$$

This paper

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$$\mathbf{A} = \begin{array}{c} \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{array} \begin{array}{c} \begin{array}{cccc} i_1 & i_2 & i_3 & i_4 \end{array} \\ \begin{pmatrix} 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{pmatrix} \end{array} \qquad \mathbf{P} = \begin{array}{c} \\ i_1 \\ i_2 \\ i_3 \\ i_4 \end{array} \begin{array}{c} \begin{array}{cccc} i_1 & i_2 & i_3 & i_4 \end{array} \\ \begin{pmatrix} 0 & 0.8 & 0.5 & 0.3 \\ 0.7 & 0 & 0.2 & 0.6 \\ 0.1 & 0.2 & 0 & 0.5 \\ 0.8 & 0.5 & 0.3 & 0 \end{pmatrix} \end{array}$$

- Observing $\mathbf{P}$ is sufficient to estimate (1).

Estimators

- Two estimators: Instrumental Variable (IV) estimator and Bayesian estimator:
  1. IV estimator: requires observation of $\bar{\mathbf{x}}_i$.
     - Make possible the use of Bramoullé, Djebbari, and Fortin 2009.
  2. Bayesian estimator: more general and does not require observing $\bar{\mathbf{x}}_i$.
     - From the likelihood of $\mathbf{y}$ conditional on $\mathbf{A}$ (L.-F. Lee 2004) to a joint-likelihood of ($\mathbf{y}$, $\mathbf{A}$) conditional on $\mathbf{P}$.
     - MCMC method to sample the unknown parameters in (1) and the network from their posterior distribution.

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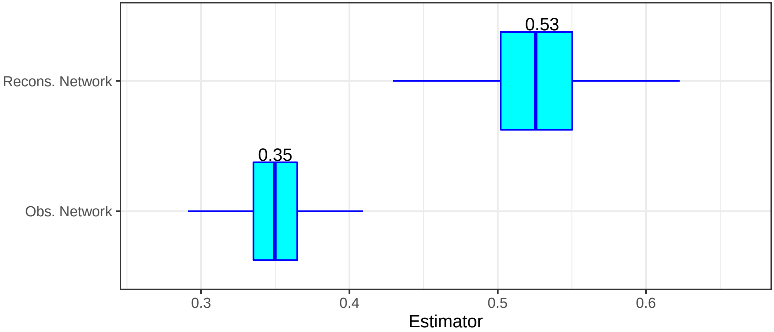
- Data: National Longitudinal Study of Adolescent to Adult Health.
- Outcome: students' academic achievement.
- Many missing links.

$$P_{ij} = \begin{cases} 1 & \text{if } i \text{ knows } j \\ \dfrac{\text{\# missing links}}{\text{\# individuals } i \text{ is not linked to}} & \text{otherwise} \end{cases}$$

Application on adolescents' academic achievements



Conclusion

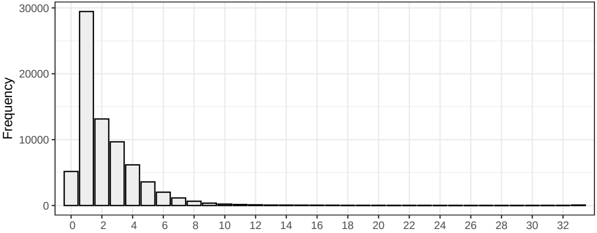
- We propose a new method to estimate peer effect when the network is not fully observed.
- We assume the network distribution is available.
- Ignoring the missing links in Add Health data has a significant impact on the peer effects estimate.
- `PartialNetwork` (R package).

Chapter 2: Count Data Models with Social Interactions under Rational Expectations

Elysée Aristide Houndetoungan

- Increasing literature network models with limited dependent outcomes:
  1. Linear-in-means model for continuous variables (Bramoullé, Djebbari, and Fortin 2009), (L.-F. Lee 2004);
  2. Binary data (Brock and Durlauf 2001; L.-f. Lee, Li, and Lin 2014);
  3. Censored choices (Xu and L.-f. Lee 2015);
  4. Multinomial choices (Brock and Durlauf 2002; Guerra and Mohnen 2020).

- Example of count data from Add Health: Number of extracurricular activities in which students are enrolled.



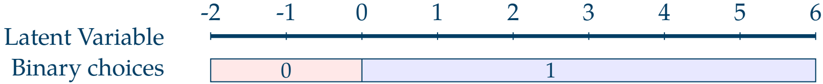
- Integer values and left censored at zero.

This paper

1. Model dealing with networks and count choices.
2. Generalization of Rational Expectation model presented by L.-f. Lee, Li, and Lin 2014 for binary outcome.
3. (Under some conditions, e.g, when the number of count choices is large) my model is asymptotically similar to the linear models.

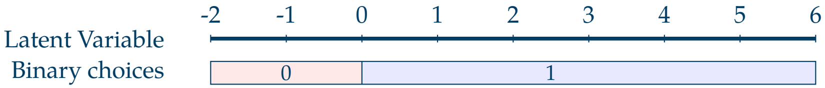
Game: Main Assumption

- Binary choices (L.-f. Lee, Li, and Lin 2014; Liu 2019).



- Assumption for count variable (see Cameron and Trivedi 1990).

- Binary choices (L.-f. Lee, Li, and Lin 2014; Liu 2019).



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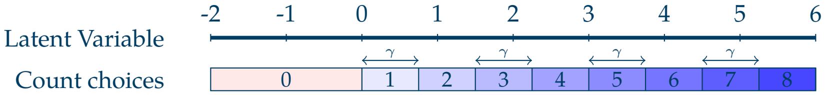
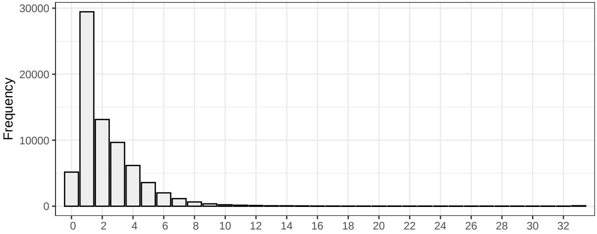


- First Order Conditions of the game resolution:

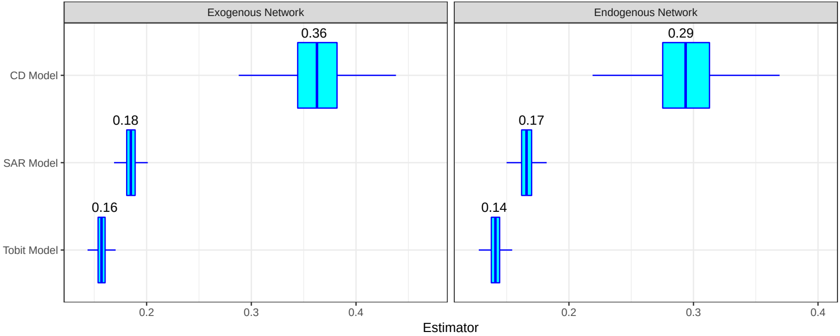
$$y_i^* = \lambda \, \mathbf{E}\left(\bar{y}_i\right) + \mathbf{x}_i'\boldsymbol{\beta} + \varepsilon_i. \tag{2}$$

- Uniqueness of the Bayesian Nash equilibrium under reasonable conditions on λ and ε_i.
- Nested Partial Likelihood (NPL) approach to estimate model parameters.

Application on students' participation in recreational activities



Application on students' participation in recreational activities



Conclusion

- Model of random utility dealing with networks and count outcome.
- The model performs well on count data.
- Two main results:
  1. Integer nature of the outcome is important;
  2. The endogeneity of the network is important.
- (Next steps:) Zeros inflated specification may be required (e.g., smoking).
- `CDatanet` (R package).

Chapter 3: Selective linear segmentation for detecting relevant parameter changes

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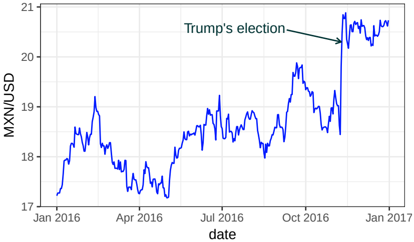
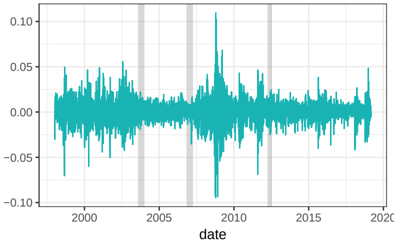
Structural Breaks

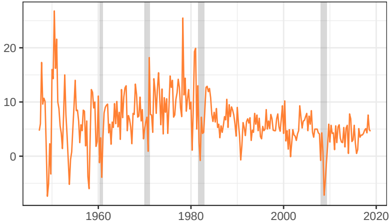
USD to Mexican Peso



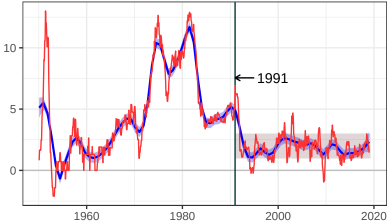
S&P 500 daily log−returns



US GDP growth rate

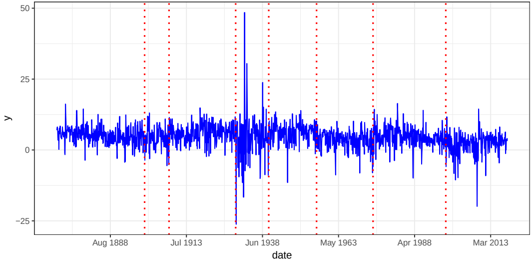


Canadian inflation (CPI)



- **Yau and Zhao's (2016) change point method (8 regimes):**

$$y_t = \beta_{0,i} + \beta_{1,i} y_{t-1} + \beta_{2,i} y_{t-2} + \varepsilon_t \quad \text{with} \quad \varepsilon_t \sim N(0, \sigma_i^2),\ t \in [\tau_{i-1}+1, \tau_i]$$



- Sequential OLS method:
  1. Over-parametrization issue;
  2. Which parameters change between two regimes;
  3. Uncertainty about parameters of short regimes.

This paper

1. We control the over-parametrization.
   - Only a small set of parameters changes over time.
2. We take model uncertainty into account.
   - We can have many model with different number of regimes in the parameters
   - A probability is associated to each model.

Method applied to S&P 500 returns

- **Results on S&P 500 returns deviation from 1871 to 2016:**

Period	Intercept	AR1	AR2
1871.02 to 1899.11	4.169	0.310	-0.078
	(0.195)	(0.024)	(0.024)
1899.12 to 1974.09	2.615	0.310	-0.078
	(0.280)	(0.024)	(0.024)
1974.10 to 1998.07	3.572	0.310	-0.078
	(0.258)	(0.024)	(0.024)
1998.08 to 2018.09	1.697	0.310	-0.078
	(0.249)	(0.024)	(0.024)

- Posterior prob of the model: 51%;
- Results that are easy to interpret;
- Less breaks in the time series;

Determine relevant breaks - Small dimension

- Test all the possibilities and choose according to a criterion (e.g. BIC).
  - Careful choice of the criterion;
  - We propose a consistent criterion that allows for comparing models in terms of probability.
- Only Works in small dimension:
  - Number of models to consider: 2^{mK};
  - **S&P500 example:** The number of models amounts to $2^{21} = 2,097,152$.

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  - **S&P500 example:** The number of models amounts to $2^{21} = 2,097,152$.

- Reframing the model with first-difference parameters,

$$\boldsymbol{\beta}_k = \boldsymbol{\beta}_{k-1} + \Delta\boldsymbol{\beta}_k.$$

- Assuming we know the true break dates, we minimize

$$\text{Objective\_function}_{OLS} + \text{Penalty\_function}(\Delta\boldsymbol{\beta}_1, \ldots, \Delta\boldsymbol{\beta}_m).$$

  Example: Lasso or Ridge penalty functions but they are biased.

- We adapt the Seemless-L0 (SELO) penalty function.

Determine relevant breaks - High dimension

- Reframing the model with first-difference parameters,

$$\boldsymbol{\beta}_k = \boldsymbol{\beta}_{k-1} + \Delta\boldsymbol{\beta}_k.$$

- Assuming we know the true break dates, we minimize

$$\text{Objective\_function}_{OLS} + \text{Penalty\_function}(\Delta\boldsymbol{\beta}_1, \ldots, \Delta\boldsymbol{\beta}_m).$$

  Example: Lasso or Ridge penalty functions but they are biased.

- We adapt the Seemless-L0 (SELO) penalty function.

Monte Carlo Study

Models		Number of regimes 1	2	3	4	True dates	Est. dates	St. dev.
DGP A	Intercept	**93**	7	0	0			
	AR_1	**97**	3	0	0			
DGP B	Intercept	**95**	5	0	0			
	AR_1	0	0	**95**	5	**[512 ; 768]**	[512.48 ; 767.64]	[6.95 ; 5.18]
	AR_2	0	**97**	3	0	**[512]**	[512.46]	[6.87]
DGP C	Intercept	**95**	5	0	0			
	AR_1	0	0	**100**	0	**[400 ; 612]**	[399.98 ; 611.98]	[7.57 ; 4.98]
DGP D	Intercept	0	**97**	3	0	**[612]**	[612.36]	[1.79]
	AR_1	0	0	**100**	0	**[400 ; 612]**	[400.19 ; 612.35]	[5.09 ; 1.77]
DGP E*	Intercept	**85**	14	1	0			
	AR_1	**89**	9	2	0			
DGP F*	Intercept	**63**	33	4	0			
	AR_1	0	27	**73**	0	**[400 ; 750]**	[394.10 ; 740.41]	[41.10 ; 54.89]
	AR_2	0	28	**72**	0	**[400 ; 750]**	[394.57 ; 745.13]	[41.29 ; 46.40]
DGP G	Intercept	0	**100**	0	0	**[351]**	[351.00]	[2.45]
	X_1	0	0	**100**	0	**[351 ; 720]**	[351.00 ; 720.05]	[2.45 ; 1.03]
	X_2	0	**100**	0	0	**[720]**	[720.05]	[1.03]

* Heteroskedastic process.

Conclusion

- Selective linear segmentation method:
  1. Detects the parameters that change from one regime to another;
  2. Shrinks every irrelevant parameters toward zero.

- Empirical contributions:
  1. Improves the interpretation of the presence of breaks;
  2. Improve model prediction performances.

- Extensions:
  1. Break in the variance;
  2. Multivariate models.

THANK YOU

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